I Semestral Exam 2002-2003
B. Math. Hons. I year
Analysis - I

Date: 04-12-2002 **Score: 100** **Instructor: T.S.S.R.K. Rao**

Answer all the questions.

1. Let $\{A_n\}$ be a sequence of countable sets. Show that $\cup A_n$ is a countable set. [10]

2. Show that $\Omega = \{p + iq : p, q \in Q\}$ is a countable dense set in **C**. [10]

3. Let A be a closed set and C a closed and bounded set in R. Show that the set $A + C = \{a + c : a \in A, c \in C\}$ is a closed set. [12]

4. Let $\sum a_n < \infty$. Suppose $a_n > 0$ for all n. Show that $\sum a_n^2 < \infty$. Give an example to show that '$a_n > 0$ for all n' cannot be dropped. [10]

5. Let $f : R \to R$ be such that for each $\alpha \in R, \{x : f(x) \geq \alpha\}$ and $\{x : f(x) \leq \alpha\}$ are both closed sets. Show that f is continuous. [12]

6. Let $f : (0, 1) \to R$ be a differentiable function such that $f' > 0$. Let $A \subseteq (0, 1)$ be such that $\bar{A} \subseteq (0, 1)$. Show that $f(\bar{A}) = \overline{f(A)}$. [12]

7. Let $f : (a, b) \to R$ be differentiable. Show that f' satisfies the intermediate value property. [10]

8. If $f : [a, b] \to R$ is a bounded and Riemann integrable function, then show that $|f|$ is also a Riemann integrable function. Give an example of a function f where $|f|$ in Riemann integrable but f is not. [12]

9. Let $f : [0, 1] \to [0, 1]$ be a continuous function such that $\int f\, dx = 0$. Show that $f(t) = 0$ for all $t \in [0, 1]$. [12]